MULTI-DIMENSIOANL LINEAR SYSTEM RELATIVE STABILITY ANALYSIS USING COMPLEX COEFFICIENTS

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ABSTRACT

In this correspondence, the relative stability analysis of multi- dimensional linear system is considered. In particular multi-dimensional (MD) characteristics equation is converted in to equivalent one dimensional (1D) characteristics equation. The relative stability analysis is done based on the damped frequency of oscillation, characteristic equations with complex coefficients arise. This complex coefficients are used in two different ways to form the Modified Routh's tables named as Sign Pair Criterions (SPC I and SPC II). The beauty of the proposed Routh's algorithm is finding the relative stability of the multi-dimensional system without determining the roots of the system. The illustrative examples shows the proposed schemes computational simplicity.

Keywords: Relative Stability, Sign Pair Criterion, Multi-dimensional (MD) system, Characteristics Equation.

1. INTRODUCTION

The information about relative stability of a control system is of paramount im-portance for any design problem. Sreekala et al [1] had proposed a stability analysis depends on damped frequency of oscillation for one dimensional linear systems us-ing sign pair criterion technique.

Hwang et al had proposed a test procedure of routh's Hurwitz stability criterion for the presence of unstable roots of a polynomial with presented complex coeffi-cients[2].Chen et al revealed the original Routh table dealing with real polynomial is next implemented for complex polynomial. The new tabular form is determined by complex polynomial with respect to imaginary axis developed [3].

Benidir et al has constructed the routh's table in complex case for vanishing lead-ing array elements without introducing the classical ε method [4].Agashe presents a new Routh algorithm for investigating the no of Right Half Plane (RHP) roots of a polynomial with a real or complex coefficients arrays. This Routh algorithm is a spe-cial case for a real polynomial [5].

Sivanandam et al had introduced the stability analysis of linear time invariant sys-tem is formulated by using Routh table. This stability criterion is directly applicable to handle the complex coefficients of the given characteristics equation [6]. Then usher had proposed the hurwitz Routh stability criteria was applied to real coefficients of algebraic polynomial, the existence of unstable roots of the polynomial is revealed in [7].

Stability analysis of the multi-dimensional system using neutral network technique is investigated by Mastorakis in [8]. The implementation procedure for stability analysis for digital filter was extended by Sivanandam et al in [9]. For stabil-ity, the real parts of all the roots of the characteristic equation must lie on the left-half of -plane. If all the coefficients are real, Routh-Hurwitz stability test is directly appli-cable as explained in [10]. If the coefficients are complex in the characteristics equa-tion, ‘Sign Pair Criterions (SPC-I, SPC-II) is formulated as given below. In this ap-proach, the elements of Routh-like table are used to formulate the stability criterion.

2. PROPOSED METHOD

2.1 Sign Pair Criterion-I [SPC-I]

With the coefficient of S^n as positive, the characteristic equation C(s) can be written as,

$$C(s) = s^n + (a_1 + jb_1)s^{n-1} + (a_2 + jb_2)s^{n-2} + \cdots + (a_k + jb_k) = 0$$

The first two rows of Routh-like table are written as shown below:

$$\begin{array}{cccccc} 1 & jb_1 & a_2 & jb_3 & a_4 & \ldots \\ a_1 & jb_2 & a_3 & jb_4 & a_5 & \ldots \end{array}$$

Applying the standard Routh multiplication rule, the subsequent elements of Routh-like table are computed and the table is computed as given below.

1	jb_1	a_2	jb_3	a_4	$...$
a_1	jb_2	a_3	jb_4	a_5	$...$
r_{31}	r_{32}	r_{34}	r_{35}	$\cdots$	
r_{41}	r_{42}	r_{43}	r_{44}	$\cdots$	
r_{51}	r_{52}	r_{53}	$\cdots$		
r_{61}	r_{62}	r_{63}	$\cdots$		
r_{71}	r_{72}	$\cdots$			
r_{81}	$\cdots$				
.	.	.	.		
.	.	.	.		

Table 1 Routh-Like Table for SPC-I

To formulate (SPC-I), the elements in the first column of Routh-like table are considered to form the pairs (P_i), $i = 1,2,\dots,n$ as depicted below:

$P_1 = (1, a_1)$ $\quad P_2 = (r_{31}, r_{41})$ $\quad P_3 = (r_{51}, r_{61})$

$P_4 = (r_{71}, r_{81})$ and so on.

2.2 Sign Pair Criterion-II (SPC-II)

The characteristic equation rewritten as

$$C(s) = s^n + (a_1 + jb_1)s^{n-1} + (a_2 + jb_2)s^{n-2} + \cdots + (a_k + jb_k) = 0$$
(1)

Substituting $s = j\omega$ in the above equation,

$$C(j\omega) = (j\omega)^n + (a_1 + jb_1)(j\omega)^{n-1} + (a_2 + jb_2)(j\omega)^{n-2} + \dots + (a_k + jb_k) = 0$$
$$= R(\omega) + jI(\omega) = 0 \qquad (2)$$

Where $R(\omega)$ is the real part and $I(\omega)$ is the imaginary part of $C(j\omega)$.

For the sake of simplicity the polynomials $R(\omega)$ and $I(\omega)$ can be written as given below.

$$R(\omega) = (A_0\omega^n + A_1\omega^{n-1} + A_2\omega^{n-2} + \cdots + A_n) \qquad (3)$$

$$I(\omega) = (B_0\omega^n + B_1\omega^{n-1} + B_2\omega^{n-2} + \cdots + B_n) \qquad (4)$$

Using the Coefficients of $R(\omega)$ and $I(\omega)$ polynomials, the second form of Routh-like table can be formulated as

Table 2 Routh-Like Table for SPC-I

2.3 Algorithm for the proposed approach

A_0	A_1	A_2	…	A_n
B_0	B_1	B_2	…	B_n
c_0	c_1	c_2	…	
d_0	d_1	d_2	…	
e_0	e_1	e_2	…	
f_0	f_1	f_2	…	
g_0	g_1	…	…	
.	.	.		
.	.	.		

1. Get the characteristic equation C(s) =0 with complex coefficients.
2. With s=jω, form C (jω)=R(ω)+j I(ω)=0.
3. Use the coefficients of R(ω) & I(ω), form the first and second rows of Routh-like table.
4. If the first element in the first row is negative, multiply the full row elements by -1.
5. If the first element in the second row is zero, interchange first and second rows and multiply all elements in the second row by -1.
6. Follow the Common Routh's multiplication rule to get the complete table with '2n+1' rows.
7. If any element of the first column starting from third, comes zero, it is replaced by a small value +0.01.
8. If all the elements in a row become zero, then the auxiliary polynomial is formed using the previous row elements and differentiated once; the coefficients of this

modified polynomial are entered instead of zeros and the table is completed by applying the Routh multiplication rule.

9. Get ‘n’ sign pairs using the first column elements starting from second row.

For forming SPC-II, the elements in the first column of Routh-Like table are considered; but A_0 is not taken into account and with $2n$ elements the sign pairs are developed as

$P_1 = (B_0, c_0) \qquad P_2 = (d_0, e_0) \qquad P_3 = (f_0, g_0) \ldots P_n.$

The given system represented by the equation (1) is stable if the sign of the each element of the pairs $P_1, P_2, P_3, \ldots, P_n$ remains the same.

3. ILLUSTRATIVE EXAMPLES

3.1 Example 1 [9]

$$C(s) = z_3z_4 + z_1z_4 + z_4 + z_2 + 5 = 0$$

Convert MD characteristics equation in to 1D characteristics equation [10]

$$C(s) = \frac{1}{z_3z_4} + \frac{1}{z_1z_4} + \frac{1}{z_4} + \frac{1}{z_2} + 5 = 0$$

Substitute $z_1 = z_2 = z_3 = z_4 = s$

$$C(s) = 5s^2 + 2s + 2 = 0$$

For a choice of α=1 and With a substitution of $C(s) = C\big(j(S + \alpha)\big)$

$C'(s) = S^2 + (2 - j0.4)S + (0.6 - j0.4) = 0$

Application of SPC-I

The Routh table is formed as

$+1$	$-j0.4$	0.6
2	$-j0.4$	
$-0.2j$	0.6	
$-6.4j$		

The sign pairs are formed as

$P_1 = (+1, +2\,)$ $\quad P_2 = (-0.2j, -6.4j)$

The Sign Pairs are $P_1 = (+1, +2\,)$ $\quad P_2 = (-0.2j, -6.4j)$ .It is noted that the two elements in each pair have the same sign and obey SPC-I. Hence the system is stable.

Application of SPC II

$$C(S) = (-\omega^2 + 0.4\omega + 0.6) + j(2\omega - 0.4) = 0$$

The Routh table is formed as

0	2	-0.4
-1	-0.4	-0.6
-2	-0.4	
-0.2	-0.6	
-6.4		

The sign pairs are formed as

$$P_1 = (-1, -2) \qquad P_2 = (-0.2, -6.4)$$

The Sign Pairs are $P_1 = (-1, -2)$ $\; P_2 = (-0.2, -6.4)$ .It is noted that the two elements in each pair have the same sign and obey SPC-II. Hence the system is stable.

Output verification using MATLAB

SPC-I

```
Input coefficients of characteristic equation,i.e:[an an-1 an-2 ... a0]= [1 2 -0.4i -0.4i 0.6]

----------------------------------------
--------The Routh-Hurwitz array is:--------

m =

   1.0000                0 - 0.4000i   0.6000
   2.0000                0 - 0.4000i        0
        0 - 0.2000i   0.6000                0
        0 - 6.4000i        0                0
   0.6000                0                0
```

Output verification Using MATLAB

SPC-II

```
Input coefficients of characteristic equation,i.e:[an an-1 an-2 ... a0]= [0 -1 2 -0.4 -0.4 -0.6]

----------------------------------------
--------The Routh-Hurwitz array is:--------

m =

        0    2.0000   -0.4000
  -1.0000   -0.4000   -0.6000
   2.0000   -0.4000         0
  -0.6000   -0.6000         0
  -2.4000         0         0
  -0.6000         0         0
```

3.2 Example [8]

$$C(s) = 0.8z_1 + 1.5z_1^2 z_2 + 1.8z_2^3 + 0.2z_3 + 1.3z_1 z_3^2 + 5.6 = 0$$

Convert MD characteristics equation in to 1D characteristics equation [10]

$$C(s) = \frac{0.8}{z_1} + \frac{1.5}{z_1^2 z_2} + \frac{1.8}{z_2^3} + \frac{0.2}{z_3} + \frac{1.3}{z_1 z_3^2} + 5.6 = 0$$

Substitute $z_1 = z_2 = z_3 = z_4 = s$

$$C(s) = 5.6s^3 + s^2 + 4.6 = 0$$

For a choice of α=1 and With a substitution of $C(s) = C\big(j(S + \alpha)\big)$

$$C'(s) = S^3 + (3 - j0.17)S^2 + (3 + j0.357)S + (1 - j0.64) = 0$$

Application of SPC-I,

The Routh table is formed

$+1$	$-0.17j$	3	$-j0.647$
3	$j0.357$	1	
$-0.289j$	2.666	$-0.64j$	
$27.3j$	-5.643		
2.59	$-0.64j$		
-12.36			

The sign pairs are formed as

$P_1 = (+1, +3)$ $P_2 = (-0.289j, +27.3j)$

$P_3 = (+2.59, -12.36)$

The Sign Pairs are

$P_1 = (+1, +3)$ $P_2 = (-0.289j, +27.3j)$ $P_3 = (+2.59, -12.36)$.

It is noted P_2&P_3 fails to obey SPC-I, then the system is unstable.

Application of SPC-II

$$C(s) = (\omega^3 + 0.17\omega^2 + 3\omega + 0.64) + j(3\omega^2 + 0.357\omega - 1) = 0$$

The Routh table is formed as

0	3	0.357	-1
-1	-0.17	-3	-0.64
3	0.357	-1	
-0.051	-3.33	-0.64	
-195.5	-38.6		
-3.319	-0.64		
-0.90			

The sign pairs are formed as

$P_1 = (-1, +3)$ $P_2 = (-0.051, -195.5)$ $P_3 = (-3.319, -0.90)$

The Sign Pairs are $P_1 = (-1, +3)$ $P_2 = (-0.051, -195.5)$ $P_3 = (-3.319, -0.90)$ .It is noted P_1 fails to obey SPC-II, then the system is unstable.

Output verification Using MATLAB

SPC-I

```
Input coefficients of characteristic equation,i.e:[an an-1 an-2 ... a0]= [1 3 -0.17i 0.357i 3 1 -0.647i]

----------------------------------------
--------The Routh-Hurwitz array is:--------

m =

   1.0000                 0 - 0.1700i   3.0000                 0 - 0.6470i
   3.0000                 0 + 0.3570i   1.0000                 0
        0 - 0.2890i   2.6667                 0 - 0.6470i       0
        0 -27.3247i  -5.7163                 0                 0
   2.7271                 0 - 0.6470i        0                 0
   0.7664                 0                  0                 0
        0 - 0.6470i       0                  0                 0
```

Output verification Using MATLAB

SPC-II

```
Input coefficients of characteristic equation,i.e:[an an-1 an-2 ... a0]= [0 -1 3 -0.17 0.357 -3 -1 -0.64

----------------------------------------
--------The Routh-Hurwitz array is:--------

m =

         0    3.0000    0.3570   -1.0000
   -1.0000   -0.1700   -3.0000   -0.6400
    3.0000    0.3570   -1.0000         0
   -0.0510   -3.3333   -0.6400         0
 -195.7214  -38.6471         0         0
   -3.3233   -0.6400         0         0
   -0.9547         0         0         0
   -0.6400         0         0         0
```

4. CONCLUSION

In this paper, The relative stability analysis of multi-dimensional linear system represented in the form of characteristics equations having complex coefficients has been performed with the help of the proposed sign pair criterion-I and sign pair criterion-II. The proposed methods are simple and direct in application compared [2] to other schemes.

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